PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Methods TEST 1 Friday 22 February 2019 TIME: 45 minutes working One-page notes allowed **Calculator Assumed** 39 marks 7 Questions

Name: Marking Key Teacher: All queries directed to Mark ONLY!

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 **(6 marks)**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	1	-2	-1
2	2	-1	1	0
3	1	-2	2	1

(a) Define $h(x) = \frac{f(x)}{g(x)}$, use the table to find the value for $h'(2)$. (3 marks)

$$h'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{[g(2)]^2}$$ ✓ Use quotient rule

$$= \frac{(-1)(1) - (2)(0)}{(1)^2}$$ ✓ substitute correct values

$$= -1$$ ✓ correct answer

(b) Define $I(x) = [g(x)]^5$, use the table to find the value for $I'(1)$. (3 marks)

$$I'(1) = 5[g(1)]^4 \times g'(1)$$ ✓ Use chain rule

$$= 5 \times (-2)^4 \times (-1)$$ ✓ substitute correct values

$$= -80$$ ✓ correct answer.

Question 2 **(3 marks)**

Find the equation of the line tangent to the function $y = (3x^2 - 2)^3$ at the point ($\cancel{2,2}$). Give your answer in the gradient-intercept form.

(2, 1000)

$$\frac{dy}{dx} = 3(3x^2 - 2)^2(6x)$$

$$x = 2, \quad \frac{dy}{dx} = 3600 \quad ✓$$

find $\frac{dy}{dx}$ at $x = 2$

$$y = 3600x + c$$

$$1000 = 7200 + c$$

$$\therefore c = -6200 \quad ✓$$

solve for constant

$$\therefore y = 3600x - 6200 \quad ✓$$

State equation of tangent.

Question 3 **(3 marks)**

If $\frac{dy}{dx} = (5x + 3)^3$, and $y = 50$ when $x = 1$, determine the expression of y in terms of x.

$$\int (5x+3)^3 \, dx$$

$$= \frac{(5x+3)^4}{4 \times 5} + c$$

$$= \frac{(5x+3)^4}{20} + c$$

$$50 = \frac{(5+3)^4}{20} + c$$

$$\therefore c = -\frac{774}{5} \text{ or } -154.8$$

$$\therefore y = \frac{(5x+3)^4}{20} - \frac{774}{5}$$

✓ Find anti-derivative with or without constant

✓ solve for constant c

✓ determine equation of y with value of constant

(Note: max 1 out of 3 if no constant used)

Question 4 **(7 marks)**

A company is purchasing a type of thin sheet metal required to make a closed cylindrical container with a capacity of 4000π cm^3. Let the radius of the cylindrical base be r and the height be h.

(a) Show that the surface area of the cylinder can be expressed as $2\pi r^2 + \dfrac{8000\pi}{r}$. (3 marks)

$h = \dfrac{V}{\pi r^2} = \dfrac{4000\pi}{\pi r^2} = \dfrac{4000}{r^2}$ ✓ determine h in terms of r

$S = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r \dfrac{4000}{r^2}$ ✓ determine S in terms of r

$= 2\pi r^2 + \dfrac{8000\pi}{r}$ ✓ Simplify

(b) Using calculus, determine the least area of metal required to make a closed cylindrical container from thin sheet metal in order that it will have a capacity of 4000π cm^3. (Work to one decimal place) (4 marks)

$S = 2\pi r^2 + 8000\pi r^{-1}$

$\dfrac{dS}{dr} = 4\pi r - \dfrac{8000\pi}{r^2}$ ✓ determine $\dfrac{dS}{dr}$

$\dfrac{dS}{dr} = 0$, $r = \sqrt[3]{2000}$ or 12.60 cm ✓ equates $\dfrac{dS}{dr} = 0$ AND Solve for r.

$\dfrac{d^2S}{dr^2} = 4\pi + \dfrac{16000\pi}{r^3} > 0.$ ✓ Use first or Second derivative to determine nature

$\therefore$ local min

$\therefore S = 2\pi(12.60)^2 + 8000\pi(12.60)^{-1}$

$= 2992.2$ cm^2 ✓ determine least S.A with units.

Question 5 **(6 marks)**

A share portfolio, initially worth \$26 000, has a value of f dollars after t months, and begins with a negative rate of growth. The rate of growth remains negative until after 20 months $(t = 20)$ when the value of the portfolio is momentarily stationary and then continues with negative growth for the life of the investment. The value of the portfolio, $f(t)$ after t months can be modelled by the following model, $f(t) = -2t^3 + bt^2 + ct + d$, $0 \le t \le 37$ months where $b, c \,\&\, d$ are constants.

Determine the values of the constants $b, c \,\&\, d$.

$f(0) = 26\,000$

$d = 26000$ ✓ determine d.

$f(t) = -2t^3 + bt^2 + ct + d$

$f'(t) = -6t^2 + 2bt + c$ ✓ determine $f'(t)$

$f''(t) = -12t + 2b$ ✓ determine $f''(t)$

$f'(20) = f''(20) = 0$ ✓ Equate first and second derivatives to 0 (identify horizontal P.O.I)

$-12(20) + 2b = 0$

$\therefore b = 120$ ✓ solve for b

$-6(20)^2 + 240(20) + c = 0$

$\therefore c = -2400$ ✓ solve for c.

Question 6 **(8 marks)**

The volume, V in cubic metres and radius R metres, of a spherical balloon are changing with time, t seconds. $V = \frac{4\pi R^3}{3}$. The radius of the balloon at any time is given by $R = 2t(t+3)^3$.

Determine the following:

a) The value of $\frac{dR}{dt}$ when $t = 1$. (3 marks)

$$\frac{dR}{dt} = 2(t+3)^3 + 2t \times 3(t+3)^2$$

$$= 2(4)^3 + 6 \times (4)^2$$

$$= 224$$

✓ Use product rule

✓ determine exp for $\frac{dR}{dt}$

✓ obtain rate at $t=1$

b) The value of $\frac{dV}{dt}$ when $t = 1$. (3 marks)

$R = 2(4)^3$

$= 128$. ✓ determine R at $t=1$

$$\frac{dV}{dt} = \frac{dV}{dR} \times \frac{dR}{dt}$$

$$= 4\pi R^2 \times (224)$$ ✓ Use chain rule

$$= 4\pi(128)^2 \times (224)$$

$$= 46118781.22$$ ✓ obtain $\frac{dV}{dt}$ at $t=1$

accept any rounding

Consider the volume of the balloon at $t = 1$.

c) Use the incremental formula to estimate the change in volume 0.1 seconds later (i.e. $t = 1.1$) (2 marks)

$$\delta V \approx \frac{dV}{dt}\, \delta t$$

$$= 46118781.22\,(0.1)$$ ✓ Use incremental formula

$$= 4611878.122$$ ✓ obtain approximate change in volume within accepted error limit $\pm 0.$

(4611878 ± 0.2)

Question 7 **(6 marks)**

The position of a train on a straight mono rail, x metres at time t seconds, is modelled by the following formula for the velocity, v in metres/second, $v = pt^2 - 12t + q$ where $p \& q$ are constants. The deceleration of the train is $8ms^{-2}$ when $t = 1$. The train has a position $x = \frac{4}{3}$ when $t = 2$ and is initially at the origin ($x = 0$).

a) Determine the values of the constants $p \& q$. (4 marks)

$a = 2pt - 12$

$-8 = 2p(1) - 12$

$p = 2.$ ✓ Solve for p using acceleration of $-8\,ms^{-2}$

$v = 2t^2 - 12t + q$

$x = \frac{2t^3}{3} - 6t^2 + qt + c$ ✓ determine displacement x.

$c = 0.$ ✓ State constant $= 0$ for x.

$\frac{4}{3} = \frac{2}{3}(2)^3 - 6(2)^2 + 2q$

$q = 10$ ✓ Determine q.

b) Determine the position of the train when the acceleration is $12ms^{-2}$. (2 marks)

$a = 4t - 12 = 12 \quad \therefore t = 6\,s.$ ✓ Determine t using acceleration

$x = \frac{2(6)^3}{3} - 6(6)^2 + 10 \times 6 = -12.$ ✓ Determine x.